



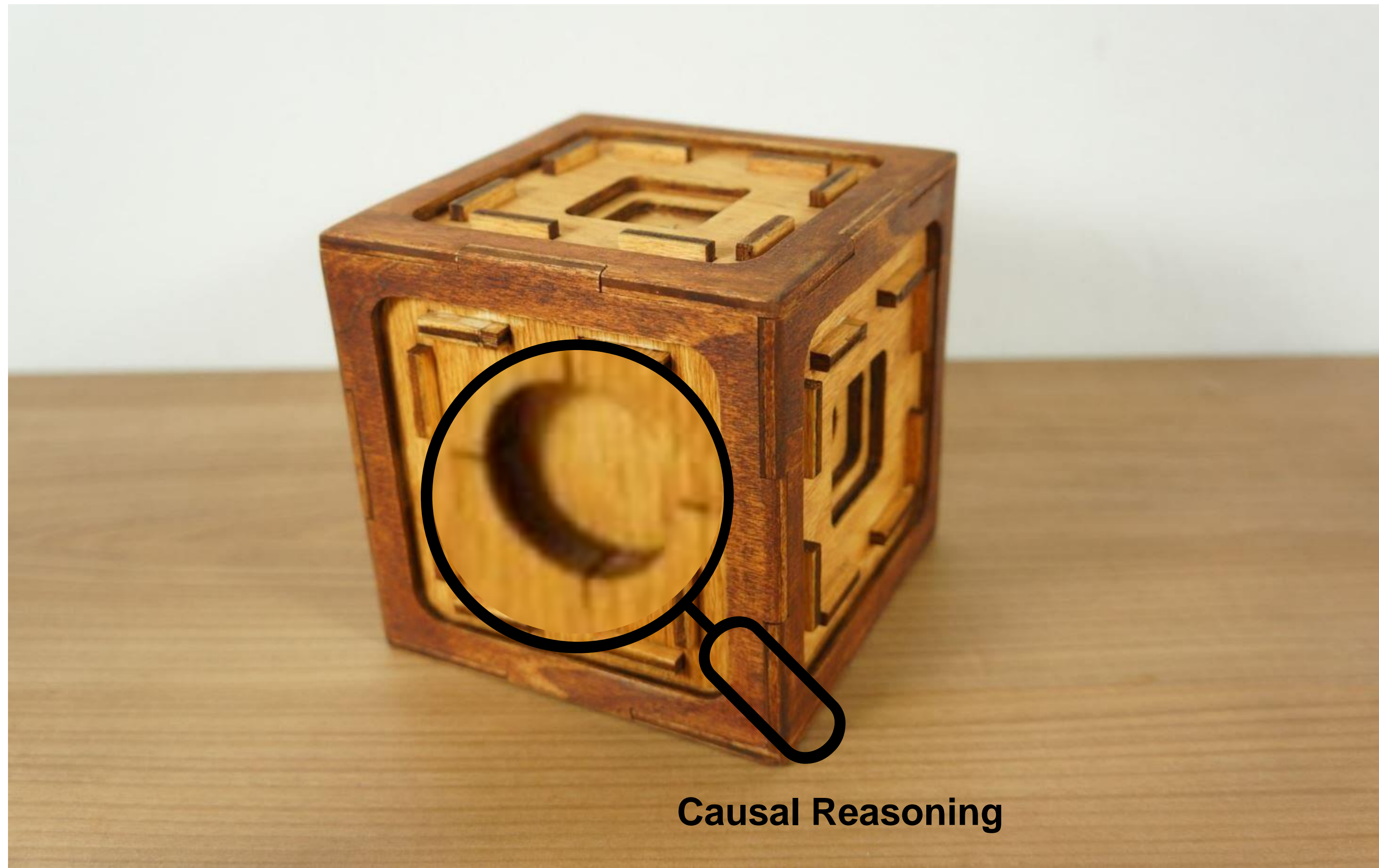
Explaining SAT Solving Using Causal Reasoning

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Behavior of SAT Solver



Causal Reasoning

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Behavior of SAT Solver

Literals blocks distance (LBD):
number of distinct decision levels of literals in a clause.

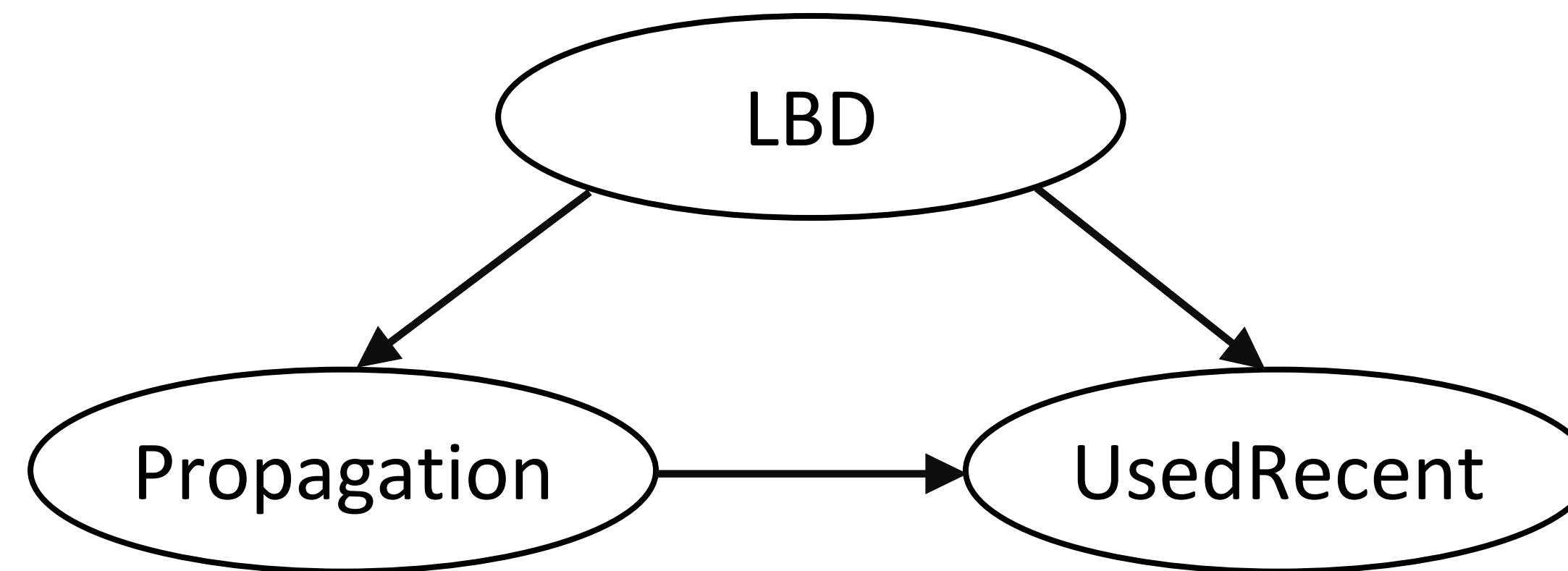
- Clauses with low LBD have greater utility.
- Small clause has greater utility.

Temperature



Causality in SAT Solver

Causal Reasoning (Pearl 2009)

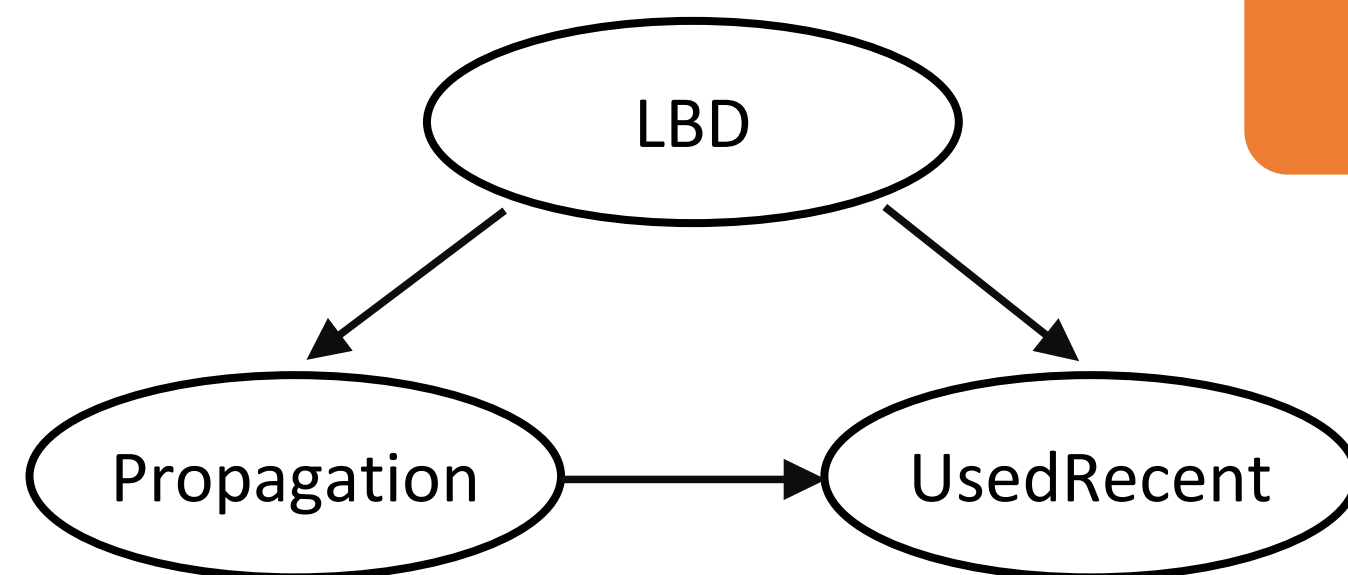


Propagation: the number of propagations the clause was involved.

UsedRecent: the number of conflicts since the clause was used in conflict analysis

Approach Overview

Branching	Size	LBD	Time	Utility
Maple	4	2	1000	10
VSIDS	7	3	10000	2
Maple	3	2	100	100



Data Generation

Structure Learning

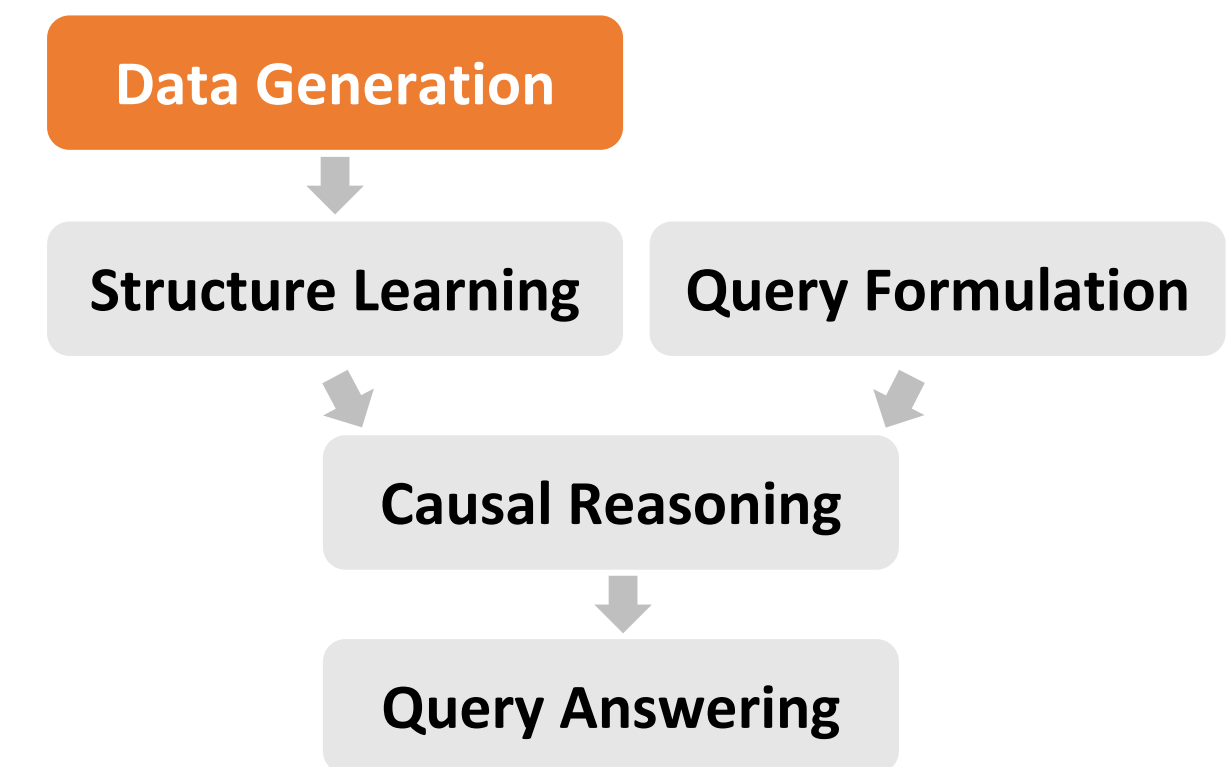
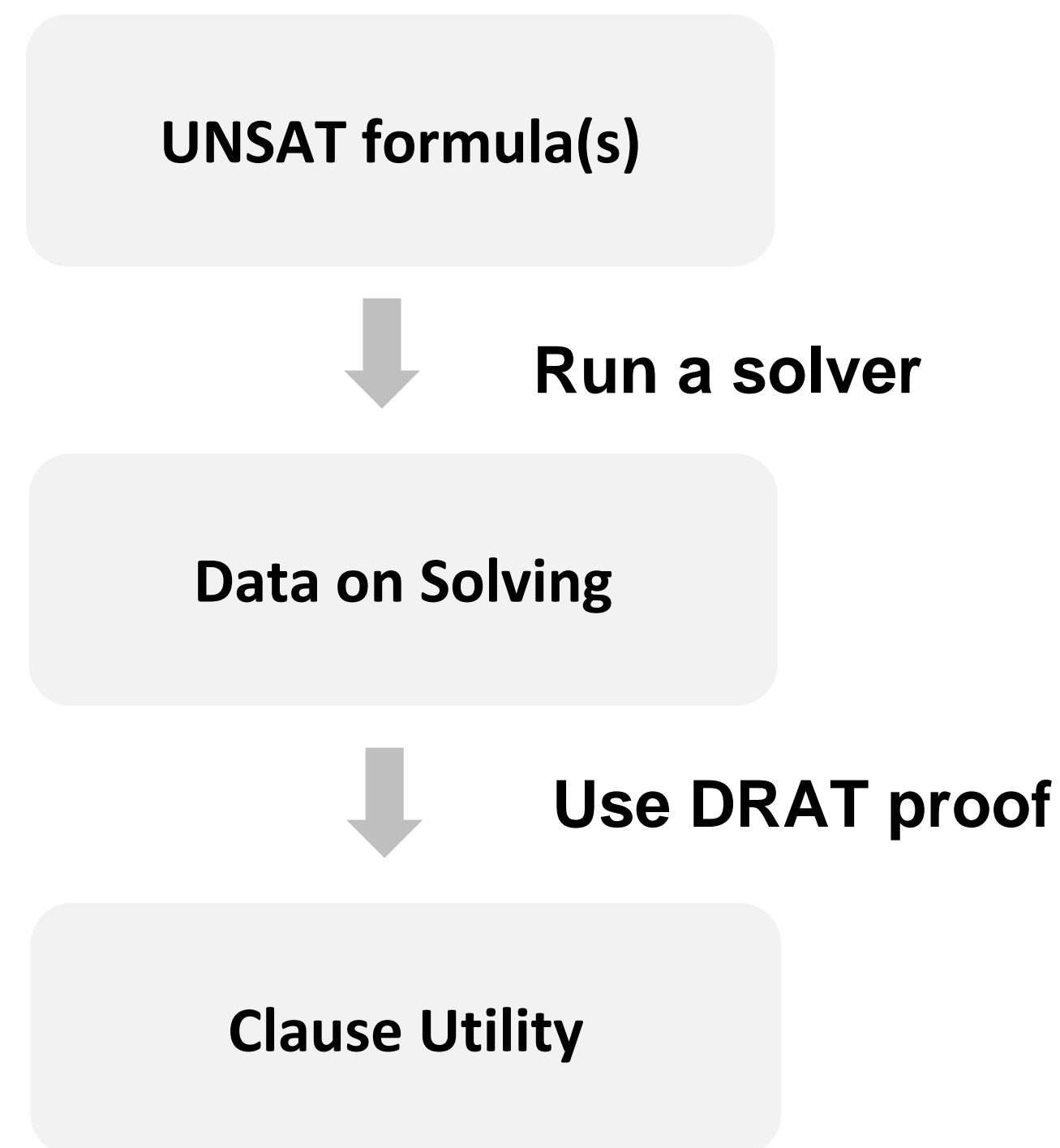
Query Formulation

Causal Reasoning

Query Answering

Does a low-LBD clause have greater utility?

Data Generation



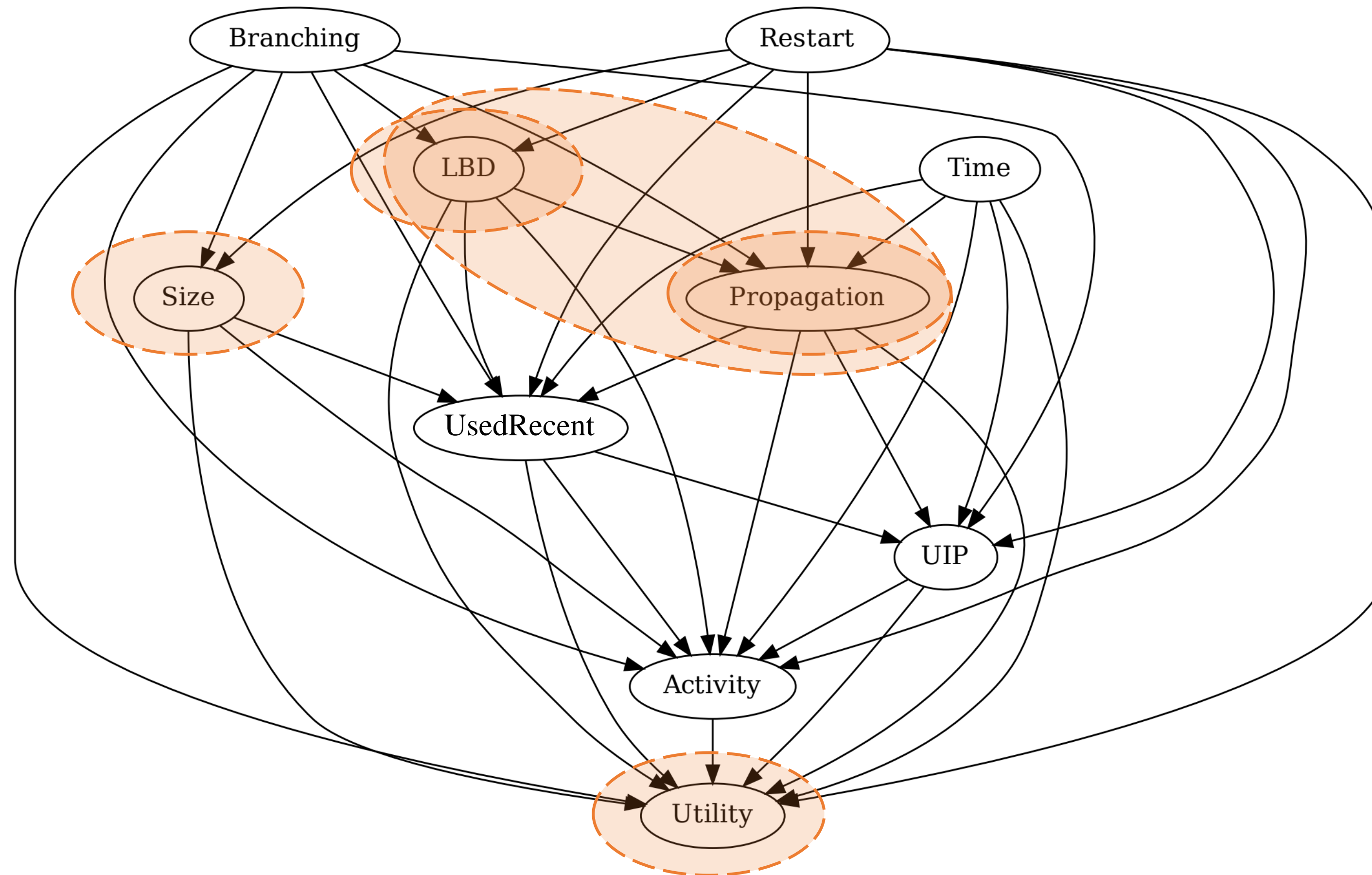
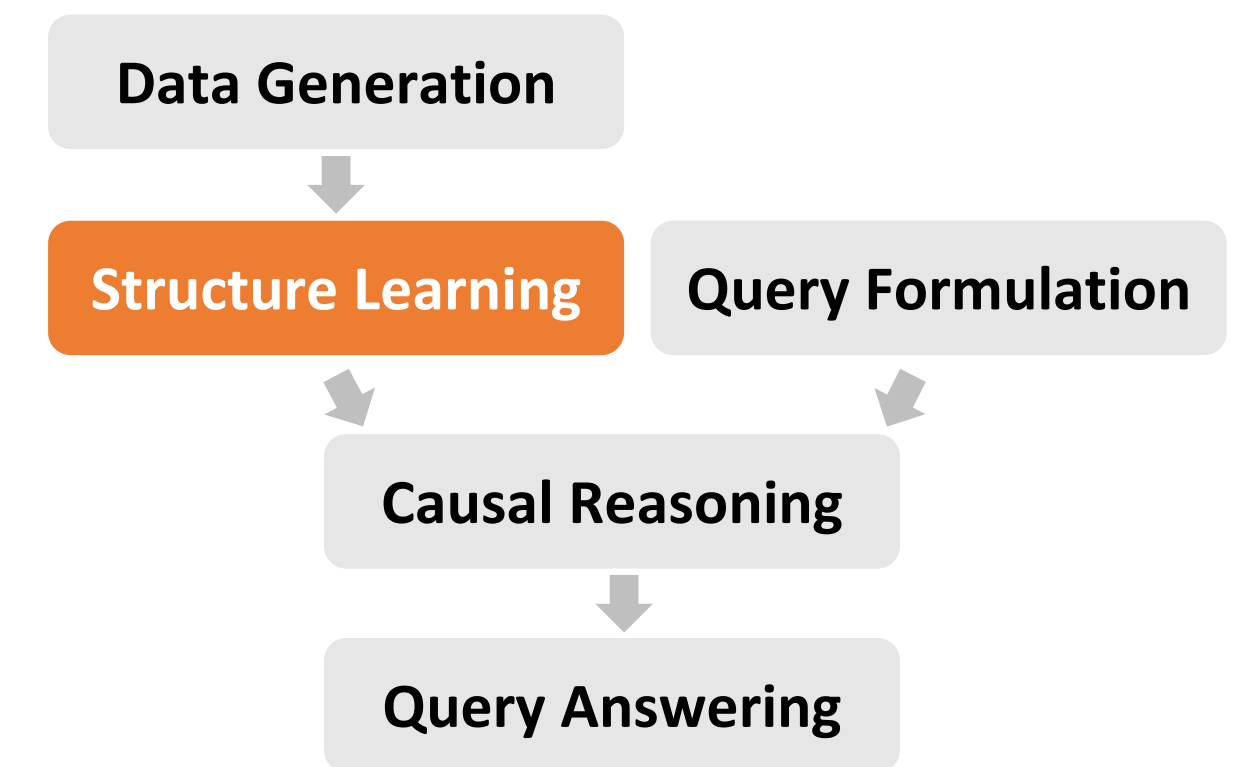
Branching	Size	LBD	Time	Utility
Maple	4	2	1000	10
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Clause Utility: the number of times a clause has been used in UNSAT proof during next 10k conflicts.

We generate data using CrystalBall Framework ^[1]

^[1] CrystalBall : Gazing at the Black-box of SAT Solving : Soos, Kulkarni, Meel (SAT '19)

Structure Learning



Causal graph

Query Formulation

Data Generation

Structure Learning

Query Formulation

Causal Reasoning

Query Answering

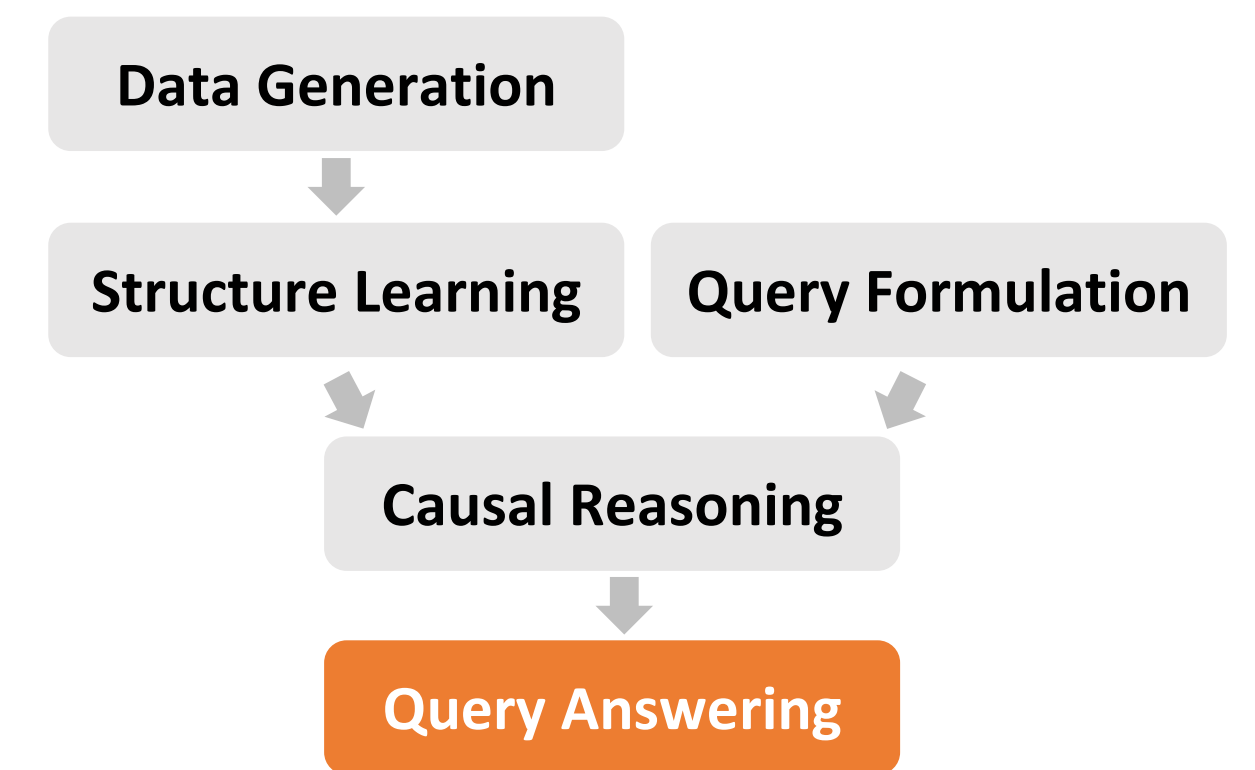
Question	Query	Explanation
Does a low-LBD clause have greater utility?	$ATE(Utility, LBD, 1) < 0$	$ATE(Y, X, a)$: effect on Y if X increases by a.
Does a clause with high LBD experience a rapid drop in utility over time?	$CATE(Utility, Time, 10000, LBD > 6) < 0$ $CATE(Utility, Time, 10000, LBD \leq 6) \geq 0$	$CATE(Y, X, a, C)$: ATE given a condition C.
Does a small clause have greater utility? What if the LBD is fixed?	$ATE(Utility, Size, 1) < 0$ $ACATE(Utility, Size, 1, LBD) > 0$	$ACATE(Y, X, a, Z)$: ATE while Z is fixed

ATE: Average Treatment Effect

CATE: Conditional Average Treatment Effect

ACATE: Average CATE

Query Answering



Question	Query	Conclusion
Does a low-LBD clause have greater utility?	$ATE(Utility, LBD, 1) = -0.26 < 0$	Low-LBD clause has greater utility.
Does a clause with high LBD experience a rapid drop in utility over time?	$CATE(Utility, Time, 10000, LBD > 6) = -0.09 < 0$ $CATE(Utility, Time, 10000, LBD \leq 6) = 0.38 > 0$	High-LBD clause experiences a rapid drop in utility over time.
Does a small clause have greater utility? What if the LBD is fixed?	$ATE(Utility, Size, 1) = -0.03 < 0$ $ACATE(Utility, Size, 1, LBD) = -0.02 < 0$	Small clause has greater utility, which also holds when LBD is fixed.

More Conclusions:

1. LBD has a greater impact than clause size.
2. Propagation has the greatest impact on utility besides LBD, size, and Activity.
3. For branching heuristic, Maple leads to greater utility than VSIDS.
4. For restart heuristic, Luby leads to the greatest utility against LBD-based and Geometric.

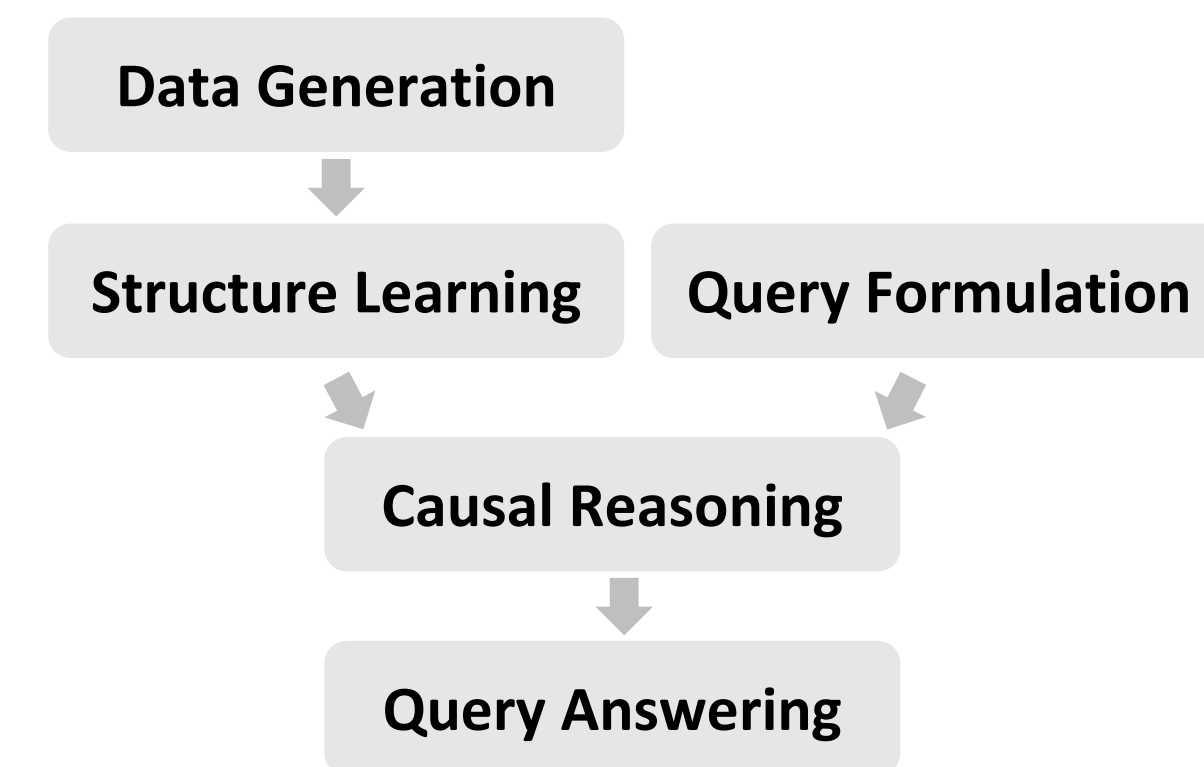
$ATE(Y, X, a)$: effect on Y if X increases by a.
 $CATE(Y, X, a, C)$: ATE given a condition C.
 $ACATE(Y, X, a, Z)$: ATE while Z is fixed.

Summary

- A framework to study the causal relationship in SAT solver.
- Causal effect on clause utility.

Future Work

- Causal effect on solving time.
- Hardness of benchmarks.



Q & A